Sph4u Energy and Momentum Answers

Work

$F = 250N$
$\theta = 45°$
$d = 11m$
$W = ?$

$W = Fd\cos\theta$
$W = (250N)(11m)\cos 45°$
$W = 1944.54\ J$
$W = 1.9 \times 10^3\ J$

Conservation of Energy -Chart

$m = 2200\ kg$
$k = 5.1 \times 10^6\ N/m$

	Before	After
h	5.00m	0.00m
Eg	mgh 107800 J	mgh 0J
X	0.00m	x=?
Ec	$\frac{1}{2}Kx^2$ 0J	$E_T - E_g - E_K$ 143704 J
v	6.7 m/s	3.5 m/s
EK	$\frac{1}{2}mv^2$ 49379J	$\frac{1}{2}mv^2$ 13475 J
ET	Eg+Ec+EK 157179 →	157179J

$x = \sqrt{\frac{2E_c}{k}}$ $x = 0.23739\ m$
$x = 0.24m$

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Conservation of Energy

$m = 2200\,kg$
$K = 5.1\times10^{6}\,N/m$
$v = 6.7\,m/s$
$v' = 3.5\,m/s$
$h = 5.00\,m$
$h' = 0\,m$
$x = 0.00\,m$
$x' = ?$

$$E_T = E_T'$$

$$E_g + E_k + E_e = E_g' + E_k' + E_e'$$

($E_e = 0$, $E_g' = 0$)

$$mgh + \frac{1}{2}mv^2 = \frac{1}{2}kx'^2 + \frac{1}{2}mv'^2$$

$$x' = \sqrt{\frac{2}{k}\left(mgh + \frac{1}{2}mv^2 - \frac{1}{2}mv'^2\right)}$$

$$x' = \sqrt{\frac{2}{5.1\times10^{6}\,N/m}\left((2200kg)(9.8\,N/kg)(5.00\,m) + \frac{1}{2}(2200\,kg)(6.7\,m/s)^2 - \frac{1}{2}(2200\,kg)(3.5\,m/s)^2\right)}$$

$x' = 0.23739\,m$

$x' = 0.24\,m$

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Linear Momentum

$\vec{p} = ?$
$m = 1.2\,kg$
$\vec{v} = 5.3\,m/s\,[N]$

$\vec{p} = m\vec{v}$
$p = (1.2\,kg)(5.3\,m/s)$
$p = 6.36\,kgm/s$

$\therefore \vec{p} = 6.4\,\frac{kgm}{s}\,[N]$

Impulse

$m = 0.430\,kg$
$\vec{v}_1 = 9.00\,m/s\,[down]$
$\vec{v}_2 = 4.23\,m/s\,[up]$
$\vec{J} = ?$

$\vec{J} = m\Delta v = m(v_2 - v_1)$

$\vec{J} = (0.430\,kg)(+4.23\,m/s - (-9.00\,m/s))$

$\vec{J} = 5.6889\,NS$

$\therefore \vec{J} = 5.69\,N\cdot s\,[up]$

Conservation of Momentum in 1-D

Forward = +ve

Before	After
$m_1 = 0.17\,kg$	$m_1' = m_1$
$v_1 = 6.4\,m/s$	$m_2' = m_2$
$m_2 = 0.16\,kg$	$v_1' = -0.125\,m/s$
$v_2 = 0\,m/s$	$v_2' = ?$

$p = p'$

$m_1v_1 + m_2v_2 = m_1'v_1' + m_2'v_2'$

$v_2' = \frac{m_1v_1 + m_2v_2 - m_1v_1'}{m_2}$

$v_2' = \frac{(0.17\,kg)(6.4\,m/s) + (0.16\,kg)(0\,m/s) - (0.17\,kg)(-0.125\,m/s)}{0.16\,kg}$

$v_2' = 6.9\,m/s\,[forward]$

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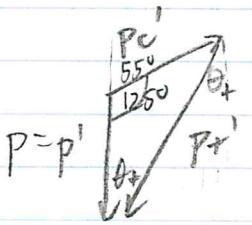
Conservation of Momentum in 2-D

Before	After	N, E +ve
$m_c = 1400\,kg$	$m_c' = m_c$	
$\vec{v}_c = 29\,m/s\ [N]$	$m_+' = m_+$	
$m_+ = 2300\,kg$	$\vec{v}_c' = 26\,m/s\ [N\ 55°\ E]$	
$\vec{v}_+ = 25\,m/s\ [S]$	$\vec{v}_+' = ?$	

$$p = m_c v_c + m_+ v_+$$

$$p = (1400\,kg)(29\,m/s) + (2300\,kg)(-25\,m/s)$$

$$p = -18300\ kg\,m/s\ [N]$$



$$p_+'^2 = p'^2 + p_c'^2 - 2p'p_c'\cos 125$$

$$p_+' = \sqrt{(18300)^2 + [(1400)(26)]^2 - 2(18300)[(1400)(26)]\cos 125}$$

$$p_+' = 49234\ \frac{kgm}{s}$$

$$\frac{\sin\theta_+}{p_c'} = \frac{\sin 125}{p_+'} \rightarrow \theta_+ = \sin^{-1}\left(\frac{m_c v_c' \sin 125}{p_+'}\right)$$

$$\theta_+ = \sin^{-1}\left(\frac{(1400kg)(26\,m/s)\sin 125}{49234}\right)$$

$$\theta_+ = 37°$$

$$p_+' = m_+ v_+' \rightarrow v_+' = \frac{p_+'}{m_+}$$

$$v_+' = \frac{49234}{2300} = 21\,m/s \quad \therefore\ 21\,m/s\ [S\ 37°\ W]$$

Sph4u Energy and Momentum Answers

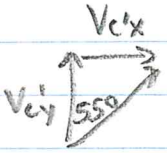
Conservation of Momentum in 2-D – Components

Before

$m_c = 1400\text{kg}$
$v_c = 28\text{m/s [N]}$
$m_t = 2300\text{kg}$
$v_t = 25\text{m/s [S]}$

After

$m_c' = m_c$
$v_c' = 26\text{ m/s [N 55° E]}$
$m_t' = m_t$
$v_t' = ?$



* E, N +ve

$p_x = p_x'$

$0 = p_{cx}' + p_{tx}' = m_c' v_{cx}' + m_t' v_{tx}' = m_c v_c' \sin\theta_c + m_t v_{tx}'$

$$v_{tx}' = \frac{-m_c v_c' \sin\theta_c}{m_t} = -\frac{(1400\text{kg})(26\text{m/s})\sin 55°}{2300\text{kg}}$$

$v_{tx}' = -12.96\text{ m/s [E]} = 12.96\text{ m/s [W]}$

$p_y = p_y'$

$m_c v_c - m_t v_t = m_c' v_{cy}' + m_t' v_{ty}' = m_c v_c' \cos\theta_c + m_t v_{ty}'$

$$v_{ty}' = \frac{m_c v_c - m_t v_t - m_c v_c' \cos\theta_c}{m_t}$$

$$v_{ty}' = \frac{(1400\text{kg})(28\text{m/s}) - (2300\text{kg})(25\text{m/s}) - (1400\text{kg})(26\text{m/s})\cos 55°}{2300\text{kg}}$$

$v_{ty}' = -17.03\text{ m/s} = 17.03\text{ m/s [S]}$

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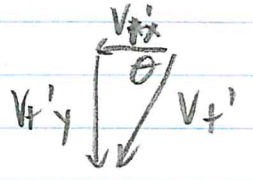
Conservation of Momentum in 2-D - components cont.

v_f':



$$v_f' = \sqrt{v_{f'x}^2 + v_{f'y}^2}$$

$$v_f' = \sqrt{(12.96\ m/s)^2 + (17.03\ m/s)^2}$$

$$v_f' = 21\ m/s$$

$$\tan\theta = \frac{v_{f'y}}{v_{f'x}}$$

$$\theta = \tan^{-1}\left(\frac{17.03}{12.96}\right)$$

$$\theta = 53°$$

$\therefore$ $v_f' = 21\ m/s\ [W\ 53°S]$

SPH4U Energy and Momentum Answers

Elastic & Inelastic Collisions

Before	After
$m_1 = 0.300\,kg$	$m_1' = m_1$
$v_1 = 2.5\,m/s$	$m_2' = m_2$
$m_2 = 0.600\,kg$	$v_1' = -0.7\,m/s$
$v_2 = 0.000\,m/s$	$v_2' = ?$

$$p = p'$$

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

$$v_2' = \frac{m_1v_1 + m_2v_2 - m_1v_1'}{m_2}$$

$$v_2' = \frac{(0.300\,kg)(2.5\,m/s) + (0.600\,kg)(0.000\,m/s) - (0.300\,kg)(-0.7\,m/s)}{0.600\,kg}$$

$$v_2' = 1.6\,m/s$$

$$E_K = E_{K1} + E_{K2}$$

$$E_K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$E_K = \frac{1}{2}(0.300\,kg)(2.5\,m/s)^2 + \frac{1}{2}(0.600\,kg)(0.000\,m/s)^2$$

$$E_K = 0.9375\,J \rightarrow 0.9\,J$$

$$E_K' = E_{K1}' + E_{K2}' = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$$

$$E_K' = \frac{1}{2}(0.300\,kg)(0.7\,m/s)^2 + \frac{1}{2}(0.600\,kg)(1.6\,m/s)^2$$

$$E_K' = 0.8415\,J \rightarrow 0.8\,J$$

$E_K \neq E_K'$ $\therefore$ Inelastic.