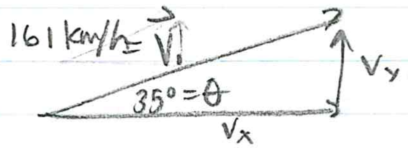
# SPH 4U Dynamics Review Answers

## Components & Projectiles

| x | y |
|---|---|
| $v_x = v_1 \cos\theta$ | $v_{y_1} = v_1 \sin\theta$ |
| $a_x = 0\ m/s^2$ | $a_y = -9.8\ m/s^2$ |
| $d_x = ?$ | $d_y = 0\ m$ |

↖ t ↗




$\vec{v}_1 = 161\ km/h$
$= 44.722\ m/s$

1. Solve for t, using y

$$d = v_1 t + \frac{1}{2} a t^2$$

$$0 = v_1 \sin\theta\, t + \frac{1}{2} a t^2$$

$$0 = t\left(v_1 \sin\theta + \frac{1}{2} a t\right)$$

$t = 0$ OR $v_1 \sin\theta + \frac{1}{2} a t = 0$

$$(44.722\ m/s)\sin 35° + \frac{1}{2}(-9.8)t = 0$$

$$25.6516 - 4.9t = 0$$

$$t = 5.235\ s$$

2. Solve for $d_x$, using x

$$d = v_1 t + \frac{1}{2} a t^2$$

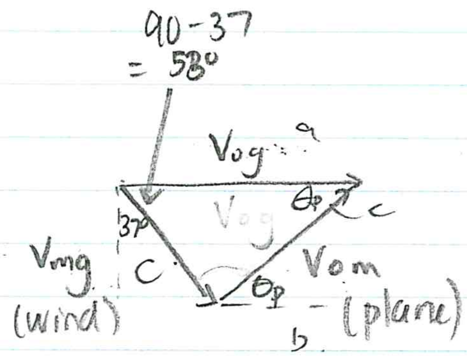
$$d = v_1 \cos\theta\, t + \frac{1}{2} a t^2$$

$$d = (44.722\ m/s)\cos 35°(5.235\ s) + \frac{1}{2}(0\ m/s^2)(5.235\ s)$$

$$d = 191.78\ m \rightarrow 1.9 \times 10^2\ m$$

# SPH 4U Dynamics Review Answers

## Relative Motion - sin Law, Cosine Law




$\theta_{plane} = ?$

$V_{mg} = 75 \text{ km/h} = 20.833 \text{ m/s}$

$V_{om} = 225 \text{ km/h} = 62.500 \text{ m/s}$

To find $\theta_p$:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin \theta_p}{V_{mg}} = \frac{\sin B}{V_{om}}$$

$$\theta_p = \sin^{-1}\left(\frac{V_{mg}\sin B}{V_{om}}\right)$$

$$\theta_p = \sin^{-1}\left(\frac{(20.833 \text{ m/s})(\sin 53^\circ)}{62.500 \text{ m/s}}\right)$$

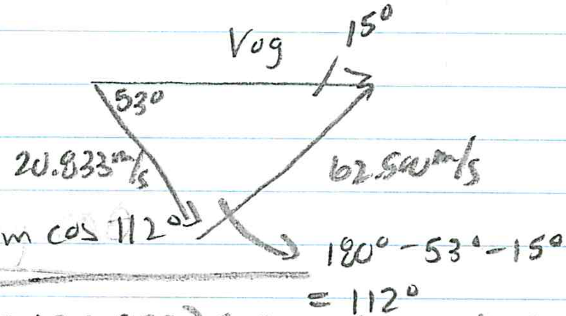
$$\theta_p = 15^\circ$$

∴ the plane must travel [E 15° N]

# SPH 4U Dynamics Review Answers

## Relative Motion con't

If you wanted to find $V_{og}$:




$$V_{og}^2 = V_{mg}^2 + V_{om}^2 - 2V_{mg}V_{om}\cos 112°$$

$$V_{og} = \sqrt{(20.833)^2 + (62.500)^2 - 2(20.833)(62.500)\cos 112°}$$

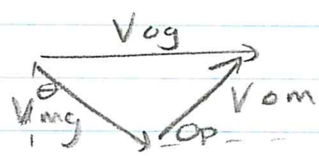
$$V_{og} = 73 \text{ m/s}$$

## Relative Motion - Components

$\vec{V}_{mg} = 20.833 \text{ m/s}$ [S 37° E]

$V_{om} = 62.500 \text{ m/s}$ $\quad \theta_p = ?$

$V_{og} =$ [E]




The vertical components of $V_{mg}$ and $V_{om}$ must be equal.

$$V_{mgy} = V_{omy}$$

$$V_{mg}\cos\theta = V_{om}\sin\theta_p$$

$$\theta_p = \sin^{-1}\left(\frac{V_{mg}\cos\theta}{V_{om}}\right)$$

$$\theta_p = \sin^{-1}\left(\frac{20.833\cos 37}{62.500}\right)$$

$\theta_p = 15°$ $\quad$ ∴ the plane must fly on the heading [E 15° N]

# SPH4U DYNAMICS REVIEW ANSWERS

Relative Motion con't:

If you wanted to find $V_{og}$:

$$V_{og} = V_{mgx} + V_{omx}$$

$$V_{og} = V_{mg} \sin\theta + V_{om} \cos\theta_p$$

$$V_{og} = (20.833\ m/s) \sin 37° + (62.500\ m/s) \cos 15.4°$$

$$V_{og} = 73\ m/s$$

# SPH 4U Dynamics Review Answers

## Newton in 2-D, slide 6

$m = 0.50$ kg
$\vec{F}_1 = 25$ N [N 30.0° E]
$\vec{F}_2 = 12$ N [S]
$\vec{F}_3 = 22$ N [N 72° E]
$a = ?$


* Not at rest, ∴ not cosine law.

1. Find Fnet

$F_{netx} = F_{1x} + F_{2x} + F_{3x}$

$F_{netx} = F_1 \sin\theta_1 + 0N + F_3 \sin\theta_3$

$F_{netx} = (25)\sin 30.0 + 0N + (22)\sin 72°$

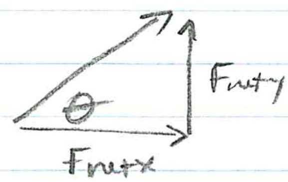
$F_{netx} = 33.42$ N [E]

$F_{nety} = F_{1y} - F_{2y} + F_{3y}$

$F_{nety} = F_1 \cos\theta_1 - F_2 + F_3 \cos\theta_3$

$F_{nety} = (25)\cos 30.0 - 12 + (22)\cos 72$

$F_{nety} = 16.45$ N [N]




$\therefore F_{net}^2 = F_{netx}^2 + F_{nety}^2$

$F_{net} = \sqrt{(33.42)^2 + (16.45)^2}$

$F_{net} = 37.25$ N

$\tan\theta = \frac{F_{nety}}{F_{netx}}$

$\theta = \tan^{-1}\left(\frac{16.45}{33.42}\right)$

$\theta = 26.21°$

# SPH4U DYNAMICS REVIEW ANSWERS

Newton in 2-D, slide 6 cont

2. Find a

$$F_{net} = ma \Rightarrow a = \frac{F_{net}}{m}$$

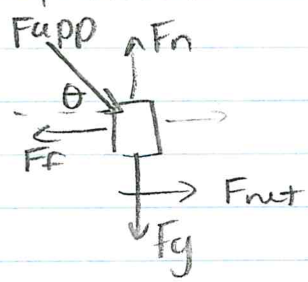
$$a = \frac{37.25}{0.50\,kg}$$

$$a = 74.5\ m/s^2$$

$$\vec{a} = 74\ m/s^2\ [E\ 26°\ N]$$

# SPH 4U DYNAMICS REVIEW ANSWERS

Newton in 2-D, slide 7




$F_{app} = 552\ N$
$\theta = 42.0°$
$m = 16.5 kg$
$\mu = 0.15$
$a = ?$

1. Find $F_n$

$\sum F_{nety} = 0N$ (because the lawnmower is not moving vertically)

$\sum F_{nety} = F_g + F_{appy} - F_n$

$F_n = F_g + F_{appy}$

$F_n = mg + F_{app} \sin\theta$

2. Find $F_{netx}$

$\sum F_{netx} = ma = F_{appx} - F_f$

$ma = F_{app} \cos\theta - \mu F_n$

$ma = F_{app} \cos\theta - \mu(mg + F_{app} \sin\theta)$

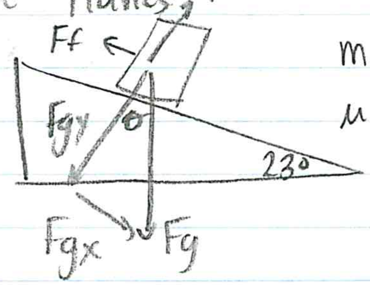
$$a = \frac{F_{app}\cos\theta - \mu mg - F_{app}\sin\theta}{m}$$

$$a = \frac{(552N)\cos 42.0° - (0.15)(16.5kg)(9.8N/kg) - (0.15)(552N)\sin 42.0°}{16.5kg}$$

$a = ~~1.0 m/s^2~~$ $20 m/s^2$.

# SPH 4U Dynamics Review Answers

Inclined Planes




m = 35 kg
μ = 0.12
θ = 23°

1. Solve for Fn

$\sum F_{net\,y} = 0\,N$

$\sum F_{net\,y} = F_n - F_{gy}$

$F_n = F_g \cos\theta = mg\cos\theta$

2. Solve for a

$F_{net\,x} = ma = F_{gx} - F_f$

$ma = F_g \sin\theta - \mu F_n$

$\cancel{m}a = \cancel{m}g\sin\theta - \mu \cancel{m}g\cos\theta$

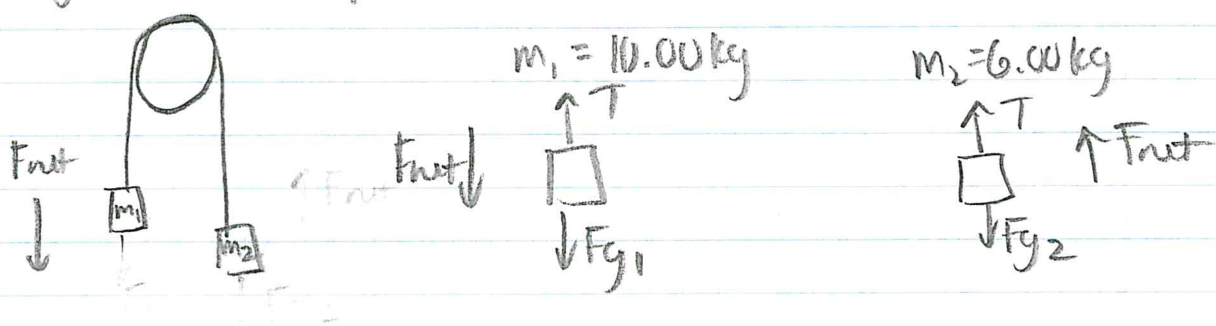
$a = g\sin\theta - \mu g\cos\theta$

$a = (9.8\,N/kg)[\sin 23° - (0.12)\cos 23°]$

$a = 2.7\,m/s^2$

# SPH 4U Dynamics Review Answers

## String and Pulley




For $m_2$:

$$F_{net} = T - F_{g2}$$
$$m_2 a = T - m_2 g$$
$$T = m_2 a + m_2 g \quad (1)$$

For $m_1$:

$$F_{net} = F_{g1} - T$$
$$m_1 a = m_1 g - T \quad (2)$$

Sub (1) into (2)

$$m_1 a = m_1 g - m_2 a - m_2 g$$
$$m_1 a + m_2 a = m_1 g - m_2 g$$
$$a = \frac{m_1 g - m_2 g}{m_1 + m_2}$$
$$a = \frac{(9.8\ N/kg)(10.00\ kg - 6.00\ kg)}{10.00\ kg + 6.00\ kg}$$

$$a = 2.45\ m/s^2 \rightarrow 2.4\ m/s^2$$

# SPH 4U DYNAMICS REVIEW ANSWERS

String and Pulley con't

To solve for T, sub a into ①

$T = m_2 a + m_2 g$

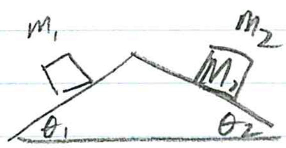
$T = (6.00\,kg)(2.45\,m/s^2) + (6.00\,kg)(9.8\,N/kg)$

$T = 73.5\,N \rightarrow 74\,N$.

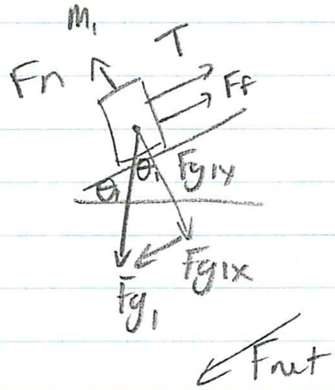
# SPH 4U Dynamics Review Answers.

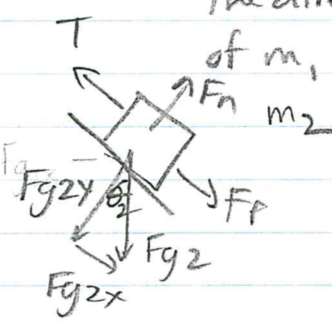
## String, Pulley & Inclined Plane.

$m_1 = 10.00\text{kg}$
$m_2 = 6.00\text{kg}$
$\mu = 0.467$
$T = ?$
$a = ?$
$\theta_1 = 50°$
$\theta_2 = 35°$




*assume Fnet is in the direction of m1




1. Find $F_n$ for $m_1$ & $m_2$

$m_1$: $\sum F_{y1} = 0N = F_{n1} - F_{g1y}$

$F_{n1} = F_{g1}\cos\theta_1 = m_1 g\cos\theta_1$

$m_2$: $\sum F_{y2} = 0N = F_{n2} - F_{g2y}$

$F_{n2} = F_{g2}\cos\theta_2 = m_2 g\cos\theta_2$

2. Fnet eq'n for $m_2$

$F_{net2} = T - F_{f2} - F_{g2x}$

$m_2 a = T - \mu F_{n2} - F_{g2}\sin\theta_2$

$m_2 a = T - \mu m_2 g\cos\theta_2 - m_2 g\sin\theta_2$

$T = m_2 a + \mu m_2 g\cos\theta_2 + m_2 g\sin\theta_2$ ①

# SPH 4U Dynamics Review Answers

String? Pulley? Inclined Plane

3. Fnet eqtn for $m_1$

$F_{net1} = F_{g1x} - T - F_{f1}$

$m_1 a = F_{g1}\sin\theta_1 - T - \mu F_{n1}$

$m_1 a = m_1 g\sin\theta_1 - T - \mu m_1 g\cos\theta_1$ ②

4. Sub ① into ②

$m_1 a = m_1 g\sin\theta_1 - m_2 a - \mu m_2 g\cos\theta_2 - m_2 g\sin\theta_2$

$m_1 a + m_2 a = g(m_1\sin\theta_1 - \mu m_2\cos\theta_2 - m_2\sin\theta_2)$

$$a = \frac{g(m_1\sin\theta_1 - \mu m_2\cos\theta_2 - m_2\sin\theta_2)}{m_1 + m_2}$$

$$a = \frac{(9.8\ \text{N/kg})(10.00\ \text{kg}\sin 50 - (0.467)(6.00)\cos 35^\circ - (6.00)\sin 35)}{10.00 + 6.00}$$

$a = 1.1783\ \text{m/s}^2 \rightarrow 1\ \text{m/s}^2$

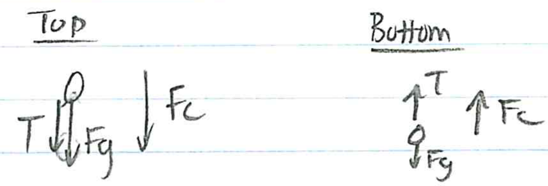
5. Sub a into ① ~~Also~~ to get T

$T = (6.00)(1.1783) + (0.467)(6.00)(9.8)\cos 35 + (6.00)(9.8)\sin 35$

$T = 63.2897\ \text{N} \rightarrow 60\ \text{N}$

# SPH 4U Dynamics Review Answers.

## Centripetal Motion




$m = 1.2\,kg$
$r = 0.80\,m$
$N = 20$
$t = 1.0\,s$
$T_{max}, T_{min} = ?$

$f = \frac{N}{t}$

$f = \frac{20}{1.0\,s}$

$f = 20\,Hz$

* T is tension.

* Remember, $F_c$ is like $F_{net}$.

1. Top.

$F_c = T + F_g$

$m4\pi^2 r f^2 = T + mg$

$T = m(4\pi^2 r f^2 - g)$

$T = (1.2\,kg)(4\pi^2(0.80\,m)(2.0\,Hz)^2 - 9.8\,N/kg)$

$T = 139.837\,N$

$T = 1.4 \times 10^2\,N \leftarrow T_{min}$

2. Bottom

$F_c = T - F_g$
$T = F_c + F_g$
$T = m4\pi^2 r f^2 + mg$
$T = (1.2\,kg)(4\pi^2(0.80\,m)(2.0\,Hz)^2 + (9.8\,N/kg))$
$T = 163.357\,N$
$T = 1.6 \times 10^2\,N \leftarrow T_{max}$

# SPH 4U Dynamics Review Answers

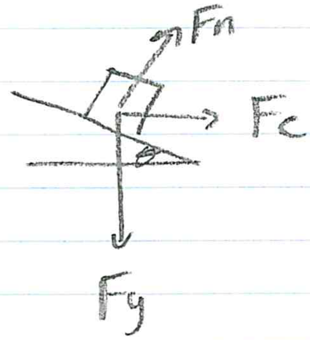
Banked Curve

$r = 800.0\,m$

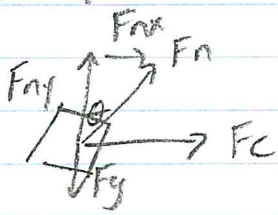
$v = 55.6\,m/s$

$\theta = ?$

1.




2. FBD in Fc




3. $\sum F_y = 0N = F_{ny} - F_g$

$F_g = F_{ny}$

$mg = F_n \cos\theta$

$F_n = \dfrac{mg}{\cos\theta}$

4. $\sum F_x = F_c = F_{nx}$

$\dfrac{mv^2}{r} = F_n \sin\theta$

$\dfrac{mv^2}{r} = \dfrac{mg}{\cos\theta}\sin\theta$ $\quad * \; \dfrac{\sin\theta}{\cos\theta} = \tan\theta$

$\dfrac{mv^2}{r} = mg\tan\theta$

# SPH 4U DYNAMICS REVIEW ANSWERS

Banked curve con't

$$\frac{v^2}{r} = g\tan\theta$$

$$\theta = \tan^{-1}\left(\frac{gv^2}{r}\right)$$

$$\theta = \tan^{-1}\left(\frac{(9.8\,N/kg)(55.6\,m/s)^2}{800.0\,m}\right)$$

$$\theta = 22^\circ$$