SPH 4U

Name: ANSWER

WINTER 2019

SPH 4U UNIT # 2 TEST – ENERGY & MOMENTUM

Knowledge & Understanding / 10 Application / 34 Thinking & Inquiry / 12

Multiple Choice (1 mark each, 10 Knowledge & Understanding Marks)

1. The units of momentum can be written as:

   a. J
   b. N·m
   c. kg·m/s²
   (d) kg·m/s
   e. None of the above

   $p = mv = \frac{kgm}{s}$

2. When does no work occur?

   a. When the object does not move → $d = 0$
   b. When the angle between the force and displacement is 0° → $\cos\theta = 1$
   c. When the angle between the force and displacement is 90° → $\cos\theta = 0$
   d. A and B are correct
   (e) A and C are correct

3. A 5.0 kg cat travelling at 1.3 m/s [E] has a momentum of:

   (a) 6.5 kg·m/s [E]
   b. 6.5 kg·m/s [W]
   c. 3.8 kg·m/s [E]
   d. 3.8 kg·m/s [W]
   e. None of the above

   $p = mv$
   $= (5.0\ kg)(1.3\ m/s)$
   $= 6.5\ \frac{kgm}{s}$

4. A bird flying at a height descends to half that height. The gravitational potential energy has changed by a factor of:

   a. 0.25
   (b) 0.5
   c. 1
   d. 2
   e. 4

   $E_g = mgh \quad E_g' = \frac{mgh}{2} = \frac{1}{2}mgh$

5. Which of the following will increase the kinetic energy of a hammer as it strikes the head of a nail?

   a. Using a hammer with greater mass
   b. Swinging the hammer with greater downward velocity
   c. Lifting the hammer to a greater height before swinging it down
   (d) All of the above
   e. None of the above

6. Suppose object A has a greater momentum than object B. Which of the following can you definitely conclude?

   $\vec{p}_A > \vec{p}_B$

   a. Object A has a greater mass than object B ✓
   b. Object A has a greater velocity than object B ✓
   (c) Object A has one or both of a greater mass and greater velocity than object B ✓
   d. All of the above
   e. None of the above can be definitely concluded

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7. An asteroid moves through deep space at a constant velocity and suddenly breaks into two pieces of equal mass. The two pieces fly off at right angles to each other. What can you conclude?

   a. The pieces have equal final speeds
   b. The pieces both travel at 45° to the original direction of the asteroid
   c. The pieces have equal kinetic energy
   d. All of the above
   e. None of the above

8. In an elastic collision:

   a. Momentum is conserved, kinetic energy is conserved
   b. Momentum is conserved, kinetic energy is not conserved
   c. Momentum is not conserved, kinetic energy is conserved
   d. Momentum is not conserved, kinetic energy is not conserved
   e. Momentum and kinetic energy do not determine whether or not a collision is inelastic

9. An object increases its velocity by a factor of 10, without changing its mass. The momentum will now be changed by a factor of:

   $p = mv \qquad p' = m(10v) = 10mv$

   a. 0.1
   b. 5
   c. 10
   d. 100
   e. The momentum will be unchanged

10. Which of the following is a scalar quantity?

   a. Energy
   b. Momentum
   c. Force
   d. All of the above
   e. None of the above

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Problems (34 Application Marks)

1. A billiard ball, mass 0.16 kg, hits a bumper. Immediately before the collision, the ball is travelling horizontally towards the bumper at 2.5 m/s. The collision lasts 0.0050 s. Immediately after the collision, the ball is travelling horizontally away from the bumper at 1.75 m/s. Find the force of the bumper on the billiard ball. ~~(5 marks)~~

Before	After
$m = 0.16\ kg$	$m' = m$
$v = 2.5\ m/s$	$v = -1.75\ m/s$ (1)

$t = 0.0050\ s$

given (1)

Forward = +ve

$\vec{J} = \Delta\vec{p}$ (1)

(1) $\vec{F}\Delta t = m(v_2 - v_1)$ (1)

$$F = \frac{m(v_2 - v_1)}{t}$$

$$F = \frac{(0.16\ kg)(-1.75\ m/s - 2.5\ m/s)}{0.0050\ s}$$

$F = -136\ N$

$F = 140\ N$ [backwards] (1)

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2. A water balloon launcher has a spring constant of 3215 N/m. To launch a 2.500 kg water balloon straight up, the launcher is stretched 0.750 m.

   a. Find the maximum height of the water balloon. HINT: at h_{max}, v = 0 m/s (7 marks)
   b. Find the speed of the water balloon when it has travelled 2.00 m. (3 marks)

a)

$m = 2.500\ kg$

$x = 0.750\ m$

$h_{max} = ?$

$v_{max} = 0\ m/s$

$x_{max} = 0\ m$

$K = 3215\ N/m$ ①

$E_T = E_T'$ ①

$E_g + E_k + E_E = E_g' + E_k' + E_E'$ ①

$E_E = E_g'$ ①

① $\frac{1}{2}Kx^2 = mgh_{max}$ ①

$h_{max} = \frac{Kx^2}{2mg}$

$h_{max} = \frac{(3215\ N/m)(0.750\ m)^2}{2(2.500\ kg)(9.8\ N/kg)}$

$h_{max} = 36.907\ m \rightarrow 37\ m$ ①

b) $h' = 2.00\ m$

$E_T = E_T'$

$E_g + E_k + E_E = E_g' + E_k' + E_E'$

$E_E = E_g' + E_k'$ ①

$\frac{1}{2}kx^2 = mgh' + \frac{1}{2}mv'^2$ ①

$v' = \sqrt{\frac{2}{m}\left(\frac{1}{2}kx^2 - mgh'\right)}$

$v' = \sqrt{\frac{2}{2.500\ kg}\left(\frac{1}{2}(3215\ N/m)(0.750\ m)^2 - (2.500\ kg)(9.8\ N/kg)(2.00\ m)\right)}$

$v' = 26.1967\ m/s \rightarrow 26\ m/s$ ①

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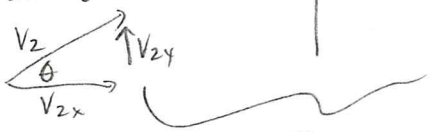
3. A vehicle, of mass 2155 kg, travelling 19 m/s [E], is struck by another vehicle, of mass 1900 kg, that is travelling 23 m/s [E 36° N]. After the collision, the vehicles are joined together.

   a) Determine the velocity of the vehicles after the collision. (13 marks)
   b) Determine whether the collision is elastic or inelastic. (6 marks)

Before	After
$m_1 = 2155\ kg$	$m' = m_1 + m_2$
$v_1 = 19\ m/s\ [E]$	$= 4055\ kg$ (1)
$m_2 = 1900\ kg$	$v' = ?$
$v_2 = 23\ m/s\ [E\ 36°\ N]$	

E, N +ve



(1)

$p_x = p_x'$ (1)

$m_1 v_{1x} + m_2 v_{2x} = m' v_x'$ (1)

$m_1 v_1 + m_2 v_2 \cos\theta = m' v_x'$ (1)

$$v_x' = \frac{m_1 v_1 + m_2 v_2 \cos\theta}{m'}$$

$$v_x' = \frac{(2155\ kg)(19\ m/s) + (1900\ kg)(23\ m/s)\cos 36°}{4055\ kg}$$

$v_x' = 18.816\ m/s\ [E]$ (1)

$p_y = p_y'$

$m_1 v_{1y} + m_2 v_{2y} = m' v_y'$ (1)

$m_2 v_2 \sin\theta = m' v_y'$ (1)

$$v_y' = \frac{m_2 v_2 \sin\theta}{m'}$$

$$v_y' = \frac{(1900\ kg)(23\ m/s)\sin 36°}{4055\ kg}$$

$v_y' = 6.334\ m/s\ [N]$

$$v = \sqrt{v_x'^2 + v_y'^2}$$ (1)

$$v = \sqrt{(18.816)^2 + (6.334)^2}$$

$v = 19.85\ m/s$ (1)

$$\tan\theta = \frac{v_y'}{v_x'}$$ (1)

$$\theta = \tan^{-1}\left(\frac{6.334}{18.816}\right)$$

$\theta = 18.6°$ (1)

$\therefore\ \vec{v} = 20\ m/s\ [E\ 19°\ N]$ (1)

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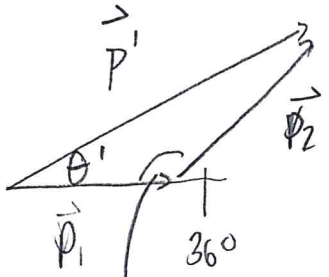
Before	After
$m_1 = 2155\,kg$	$m' = m_1 + m_2$ (1)
$\vec{v}_1 = 19\,m/s\,[E]$	$v' = ?$
$m_2 = 1900\,kg$	
$\vec{v}_2 = 23\,m/s\,[E36°N]$	

(1)



(1)

$\theta = 90 + (90 - 36)$

$\theta = 144°$ (1)

$p'^2 = p_1^2 + p_2^2 - 2p_1p_2\cos\theta$ (1)

(1)

$(m'v')^2 = (m_1v_1)^2 + (m_2v_2)^2 - 2m_1v_1m_2v_2\cos\theta$

$$v' = \sqrt{\frac{m_1^2v_1^2 + m_2^2v_2^2 - 2m_1v_1m_2v_2\cos\theta}{m'^2}}$$

$$v' = \sqrt{\frac{(2155kg)^2(19m/s)^2 + (1900kg)^2(23m/s)^2 - 2(2155kg)(19m/s)(1900kg)(23m/s)\cos 144}{(1900kg + 2155kg)^2}}$$

$v' = 19.95\,m/s$ (1)

$\frac{\sin\theta'}{p_2} = \frac{\sin\theta}{p'}$ (1)

$\theta' = \sin^{-1}\left(\frac{m_2v_2\sin\theta}{m'v'}\right)$ (1)

$$\theta' = \sin^{-1}\left(\frac{(1900kg)(23m/s)\sin 144°}{(1900kg + 2155kg)(19.85m/s)}\right)$$

$\theta' = 18.6°$ (1)

$\therefore \vec{v}' = 20\,m/s\,[E19°N]$ (1)

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b) $E_K \stackrel{?}{=} E_K'$

$E_K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$ (1) (1)

$E_K = \frac{1}{2}(2155\text{kg})(19\text{m/s})^2 + \frac{1}{2}(1900\text{kg})(23\text{m/s})^2$

$E_K = 891527.5\text{J}$

$E_K = 890000\text{J}$ (1)

$E_K' = \frac{1}{2} m' v'^2$

$E_K' = \frac{1}{2}(1900\text{kg} + 2155\text{kg})(19.85\text{m/s})^2$

$E_K' = 798880.6\text{J}$

$E_K' = 800000\text{J}$ (1)

$E_K \neq E_K'$ (1)

$\therefore$ inelastic (1)

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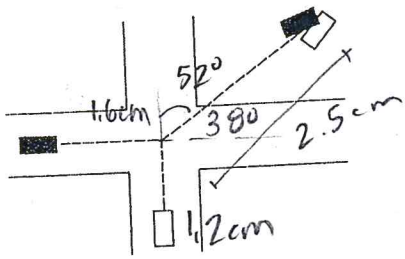
Making Connections (12 Thinking & Inquiry Marks)

1. The police report of an accident between two cars at an intersection contains the scale diagram shown, and describe the cars as identical models (having the same mass), and the road as icy. Which car was travelling faster at the moment of impact, and how much faster was it travelling? Take measurements from the diagram as you feel is necessary. (4 marks)



$\vec{p}' \propto 2.5$

$\therefore p_y' = 2.5\cos 52°$ $\quad p_x' = 2.5\sin 52$

$p_y' = 1.5$ $\quad p_x' = 2.0$

$p_x' > p_y'$

White car only in y, Black car only in x

$\therefore$ p_x' is all from black car

p_y' is all from white car

$p_x' = mv_x'$ $\quad p_y' = mv_y'$

Assuming m to be equal, for

$p_x' > p_y'$, $p_x > p_y$, $v_x > v_y$

$\therefore v_x > v_y$, black car faster

How many times?

$v \rightarrow \frac{1.6}{1.2} = 1.3x$ $\quad v' \rightarrow \frac{2.0}{1.5} = 1.3x$

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2. Derive the Work-Energy equation: $W = \Delta E_k$. Begin with the formula for work, $W = F\Delta d$. (4 marks)

$W = F\Delta d$

But, $F = ma$

$\therefore W = ma\Delta d$

But $v_2^2 = v_1^2 + 2a\Delta d \rightarrow a\Delta d = \frac{v_2^2 - v_1^2}{2}$

$\therefore W = \frac{m(v_2^2 - v_1^2)}{2}$

$W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

$\therefore W = E_{K_2} - E_{K_1}$

$\therefore W = \Delta E_K$

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3. Using concepts from this unit, explain why 'follow through' is important in sports. Choose one specific example to use when explaining your answer. (4 marks)

Sport ①

$\vec{J} = F\Delta t$ → gives greater Δt, results in greater $\vec{J}$

Greater $\vec{J} = \Delta\vec{p}$

$\Delta p = mv_2 - mv_1$

Greater Δp = greater v_2 (assuming m is constant).