Elastic Potential Energy

HOOKE'S LAW

An elastic body is one that can be deformed by a force to store energy and will transfer the energy when it returns to its state of rest.

Examples of elastic bodies

- elastics
- springs rubber
- ligaments

The spring constant shows the “spring-i-ness” of the elastic body.

The greater the spring constant, the more difficult it is to deform the elastic body.

Hooke’s Law states that the deformation of an elastic body is proportional to the force that deforms it.

$$\boxed{\vec{F} = k\vec{x}}$$

Where F is the force in N

k is the spring constant in N/m

x is the extension or compression in m

Ex. Determine the spring constant of an elastic if it takes 10N to stretch it 0.25 m.

$F = 10N$

$x = 0.25m$

$k = ?$

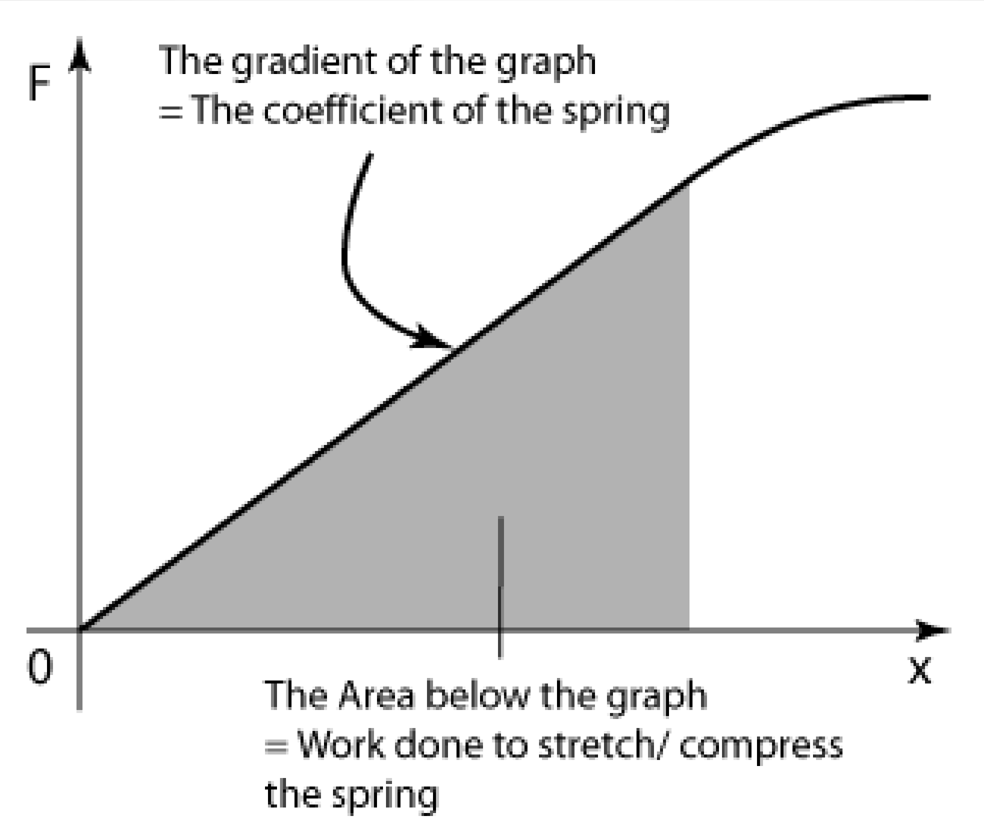
$F = kx$

$K = \frac{F}{x}$

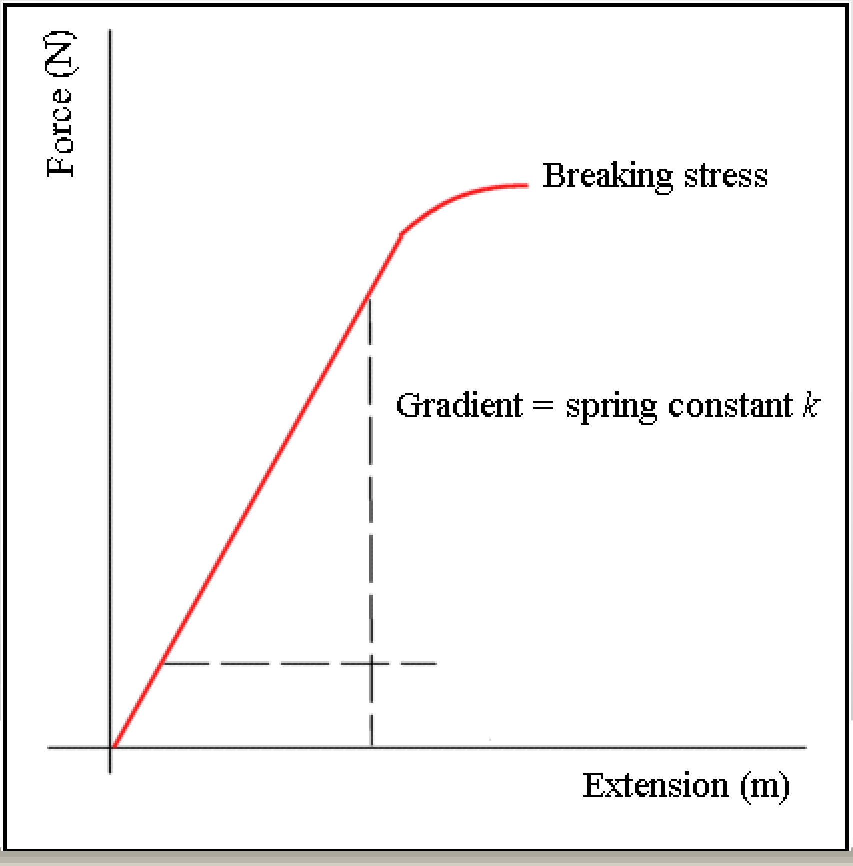
$K = \frac{10\ N}{0.25\ m}$

$K = 40\ N/m$

The spring constant can also be found by calculating the slope of the F vs. x graph in the elastic region.



Once so much force is applied to an elastic body that it will no longer return to its original state, it is permanently deformed or it has reached its elastic limit and is now inelastic.



- The elastic potential energy is stored energy in an elastic body.
- If the elastic body is stretched or compressed, it can do work returning to its state of rest.

$$\boxed{E_{E} = \tfrac{1}{2}\, kx^2}$$

E_{E} is the elastic potential energy in J

k is the spring constant in N/m

x is the extension/compression in m

$$\frac{kx^2}{2}$$

Example 1: A 40 kg child jumps from a height of 0.25 m and lands on her bed. If the mattress has a spring constant of 680 N/m, how much will the spring deform?

$m = 40 kg$

$h = 0.25 m$

$k = 680 N/m$

$\Delta x = ?$

Before: $E_T = E_g + \underbrace{E_K}_{0 J} + \underbrace{E_E}_{0 J}$

$E_T = E_g = mgh$

After: $E_T' = \underbrace{E_g'}_{0 J} + \underbrace{E_K'}_{0 J} + E_E'$

$E_T' = E_E'$

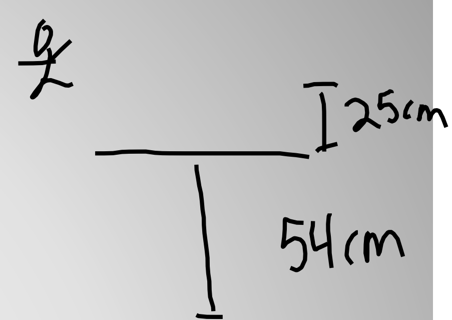
$$E_T = E_T'$$

$$mgh = \frac{1}{2}kx^2$$

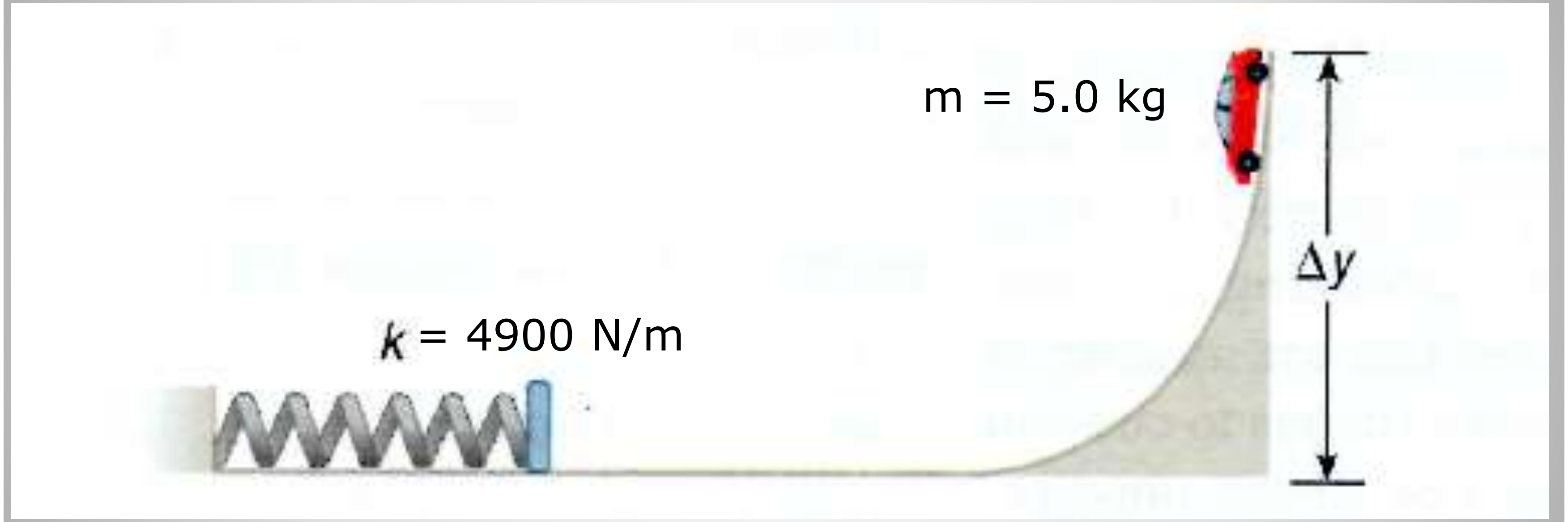
$$\sqrt{\frac{2mgh}{k}} = x$$

$$\sqrt{\frac{2(40kg)(9.8 m/s^2)(0.25m)}{680 N/m}} = x$$

$$x = 0.54 m$$



Example 2: Examine the diagram below.



a) The spring experiences a maximum compression of 22 cm. Determine the height of the initial release point.

$x = 22cm$

$x = 0.22m$

$k = 4900 N/m$

$m = 5.0kg$

$h = ?$

Release

Spring

$E_T = E_T'$

$E_g + \cancel{E_k}^0 + \cancel{E_E}^0 = \cancel{E_g'}^0 + \cancel{E_k'}^0 + E_E'$

$E_g = E_E'$

$mgh = \frac{1}{2}kx^2$

$h = \frac{kx^2}{2mg} = \frac{(4900 N/m)(0.22m)^2}{2(5.0kg)(9.8 N/kg)}$

$h = 2.4m$

b) Calculate the speed of the model car when the spring has been compressed 15 cm.

Drop ' Spring

$v = ?$

$E_T = E_T'$

$E_g + \cancel{E_k}^0 + \cancel{E_E}^0 = \cancel{E_g'}^0 + E_k' + E_E'$

$E_g = E_k' + E_E'$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

$mgh - \frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$\sqrt{\frac{2}{m}\left(mgh - \frac{1}{2}kx^2\right)} = v = \sqrt{\frac{2}{5.0}\left[(5.0)(9.8)(2.4) - \frac{1}{2}(490)(0.15)^2\right]}$

$5.0\,m/s$

c) Determine the maximum acceleration of the car after it hits the spring.

$$F_{net} = F_{spring}$$

$$ma = -kx$$

$$a = \frac{kx}{m}$$

$$a = \frac{(4900\,N/m)(0.22\,m)}{5.0\,kg}$$

$$a = 220\,m/s^2\ [\text{Back}]$$