2-D Momentum Conservation

March 29th, 2019.

Ex 1.

$m_1 = m_2 = m$

$\vec{v}_1 = 20\ m/s\ [E]$

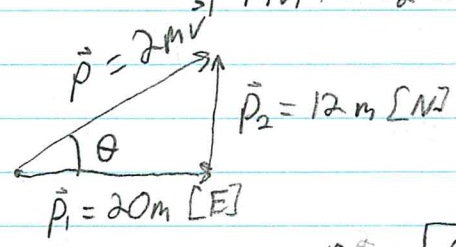
$\vec{v}_2 = 12\ m/s\ [N]$

$\vec{p} = m\vec{v}_1 + m\vec{v}_2$

$\vec{p}' = 2m\vec{v}'$

$\vec{p} = \vec{p}'$

$m\vec{v}_1 + m\vec{v}_2 = 2m\vec{v}'$



$\theta = \tan^{-1}\left(\frac{12m}{20m}\right)$

$= 31^\circ$

$p^2 = \sqrt{p_1^2 + p_2^2}$

$p = \sqrt{(20m)^2 + (12m)^2}$

$= \sqrt{400m^2 + 144m^2}$

$= \sqrt{544m^2}$

$p = 23.32m$

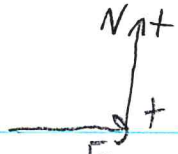
$\vec{p} = 23.32m\ [E\ 31^\circ N]$

$\vec{p} = 2m\vec{v}'$

$\vec{v}' = \frac{23.32m}{2m}\ [E\ 31^\circ N]$

$= 11.65\ m/s\ [E\ 31^\circ N]$

Ex 2.



$m_1 = 1100$ kg

$\vec{v}_1 = 25$ m/s [E 30° N]

$m_2 = 850$ kg

$\vec{v}_1' = 15$ m/s [N 10° W]

$v_2' = 20$ m/s [E 70° S]

Split into x and y.

$\vec{v}_1$

$v_{1y} = v_1 \sin 30°$

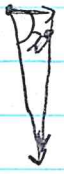
$v_{1x} = v_1 \cos 30°$

$\vec{v}_1'$

$v_{1x}' = -v_1' \sin 10°$

$v_{1y}' = v_1' \cos 10°$

$\vec{v}_2'$



$v_{2x}' = v_2 \cos 70°$

$v_{2y}' = -v_2 \sin 70°$

X

$p_x = m_1 v_{1x} + m_2 v_{2x}$ $\quad p_x' = m_1 v_{1x}' + m_2 v_{2x}'$

$p_x = p_x'$ $\quad m_1 v_{1x} + m_2 v_{2x} = m_1 v_{1x}' + m_2 v_{2x}'$

$$v_{2x} = \frac{m_1 v_{1x}' + m_2 v_{2x}' - m_1 v_{1x}}{m_2}$$

$$= \frac{-1100(15 \sin 10°) + 850(20 \cos 70°) - 1100(25 \cos 30°)}{850}$$

$v_{2x} = -24.5$ m/s

Y

$p_y = m_1 v_{1y} + m_2 v_{2y}$ $\quad p_y' = m_1 v_{1y}' + m_2 v_{2y}'$

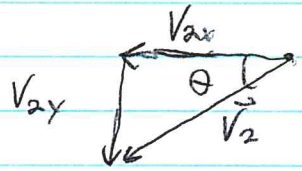
$p_y = p_y'$ $\quad m_1 v_{1y} + m_2 v_{2y} = m_1 v_{1y}' + m_2 v_{2y}'$

$$v_{2y} = \frac{m_1 v_{1y}' + m_2 v_{2y}' - m_1 v_{1y}}{m_2}$$

$$= \frac{1100(15 \cos 10°) - 850(20 \sin 70°) - 1100(25 \sin 30°)}{850}$$

$v_{2y} = -15.9$ m/s

$\vec{v}_2 = \vec{v}_{2x} + \vec{v}_{2y}$



$\theta = \tan^{-1}\left(\frac{v_{2y}}{v_{2x}}\right)$

$= \tan^{-1}\left(\frac{15.9}{24.5}\right)$

$= 33°$

$v_2 = \sqrt{v_{2x}^2 + v_{2y}^2}$

$= \sqrt{24.5^2 + 15.9^2}$

$v_2 = 29.2$ m/s

$\vec{v}_2 = 29.2$ m/s [W 33° S]

Ex 3

$m_1 = m$

$m_2 = 2m_1 = 2m$

$m_3 = \frac{m_2}{3} = \frac{2m}{3}$

$\vec{V}_1' = 9.0\ m/s\ [N\ 5.0^\circ E]$

$\vec{V}_2' = 7.5\ m/s\ [N\ 85^\circ W]$

$\vec{P}_1' = 9.0m\ [N\ 5.0^\circ E]$

$\vec{P}_2' = 15.0m\ [N\ 85^\circ W]$

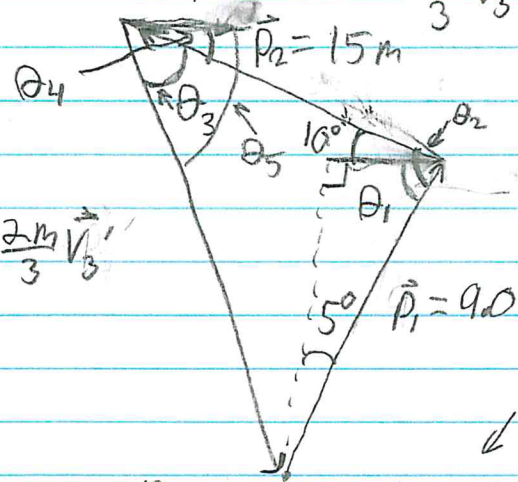
$\vec{P} = 0$ (stationary)

$\vec{P}' = m_1\vec{V}_1' + m_2\vec{V}_2' + m_3\vec{V}_3'$

$= m\vec{V}_1' + 2m\vec{V}_2' + \frac{2m}{3}\vec{V}_3'$

$\vec{P} = \vec{P}'$

$m\vec{V}_1' + 2m\vec{V}_2' + \frac{2m}{3}\vec{V}_3' = 0$



$\vec{P}_3' = \frac{2m}{3}\vec{V}_3'$

$180 = 90^\circ + 5^\circ + \theta_1$

$\theta_1 = 85^\circ$

$\theta_2 = \theta_1 + 10^\circ = 95^\circ$

Cosine law

$P_3'^2 = P_1'^2 + P_2'^2 - 2P_1'P_2'\cos\theta_2$

$= (9m)^2 + (15m)^2 - 2(9m)(15m)\cos 95^\circ$

$= 81m^2 + 225m^2 - 270m^2\cos 95^\circ$

$= 329.5\ m^2$

$P_3' = 18.2m$

Sine law

$\frac{\sin\theta_3}{P_1'} = \frac{\sin\theta_2}{P_3'}$

$\theta_3 = \sin^{-1}\left(\frac{\sin\theta_2}{P_3'}P_1'\right)$

$= \sin^{-1}\left(\frac{\sin 95^\circ}{18.2m}\ 9.0m\right)$

$= 29.51^\circ$

By Z rule, $\theta_4 = 10^\circ$

$\theta_5 = \theta_3 + \theta_4$

$= 29.51^\circ + 10^\circ$

$\theta_5 = 39.51^\circ$

$\vec{P}_3' = 18.2m\ [E\ 39.51^\circ S]$

$\vec{P}_3' = \frac{2m}{3}\vec{V}_3'$

$\vec{V}_3' = \frac{3\vec{P}_3'}{2m}$

$= \frac{3(18.2m)}{2m}\ [E\ 39.51^\circ S]$

$\vec{V}_3' = 27.3\ m/s\ [E\ 39.51^\circ S]$