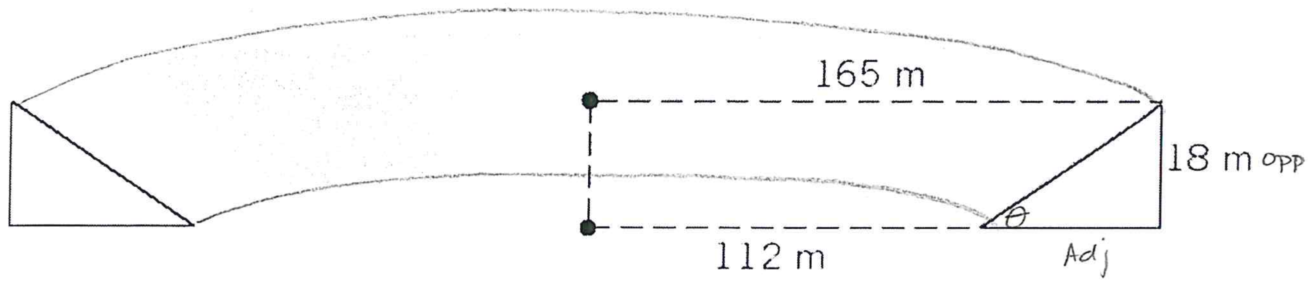
BANKED CURVES PROBLEM

1. On a banked race track, the smallest circular path on which cars can move has a radius of 112 m, while the largest has a radius of 165 m, as the drawing below illustrates. The height of the outer wall is 18 m. Find the smallest and largest speed at which cars can move on this track, if the coefficient of friction between the tires and the track is 0.900.



1. Find θ

$Adj = 165m - 112m$

$= 53m$

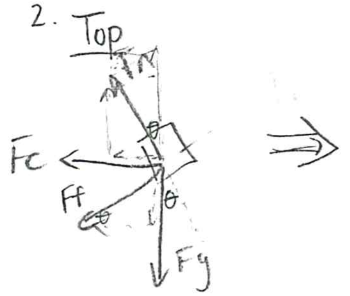
$\tan\theta = \frac{opp}{Adj}$

$\theta = \tan^{-1}\left(\frac{18}{53}\right)$

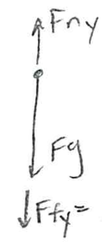
$\theta = 18.759°$

$\theta = 18.76°$

2. Top



3. Y



$\Sigma F_{net y} = 0N = F_g + F_{fy} - F_{ny}$

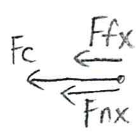
$F_{ny} = mg + \mu F_n \sin\theta$

$F_n\cos\theta = mg + \mu F_n \sin\theta$

$F_n\cos\theta - \mu F_n\sin\theta = mg$

$F_n = \frac{mg}{\cos\theta - \mu\sin\theta}$

4. X



$\Sigma F_x = F_c = F_{fx} + F_{nx}$

$\frac{mv^2}{r} = F_f\cos\theta + F_{nx}$

$\frac{mv^2}{r} = \mu F_n\cos\theta + F_n\sin\theta$

$\frac{mv^2}{r} = F_n(\mu\cos\theta + \sin\theta)$

$\frac{mv^2}{r} = \frac{mg}{\cos\theta - \mu\sin\theta}(\mu\cos\theta + \sin\theta)$

$$\frac{v^2}{r} = \frac{g(\mu\cos\theta + \sin\theta)}{\cos\theta - \mu\sin\theta}$$

$$v = \sqrt{\frac{gr(\mu\cos\theta + \sin\theta)}{\cos\theta - \mu\sin\theta}}$$

$$v = \sqrt{\frac{(9.8)(165)[(0.900)\cos 18.76 + \sin 18.76]}{(\cos 18.76 - (0.900)\sin 18.76)}}$$

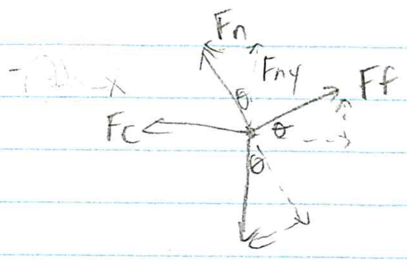
$v = 53.73\ m/s$

$v = 54\ m/s.$

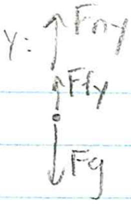
Bottom

$v = 44.2660\ m/s$

$v = 44\ m/s.$



Y:



$$\sum F_y = 0N = F_g - F_{ny} - F_{fy}$$

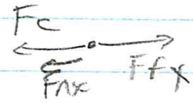
$$0 = mg - F_n\cos\theta - F_f\sin\theta$$

$$0 = mg - F_n\cos\theta - \mu F_n\sin\theta$$

$$F_n(\cos\theta + \mu\sin\theta) = mg$$

$$F_n = \frac{mg}{\cos\theta + \mu\sin\theta}$$

X:



$$\sum F_x = F_c = F_{nx} - F_{fx}$$

$$\frac{mv^2}{r} = F_n\sin\theta - F_f\cos\theta$$

$$\frac{mv^2}{r} = F_n\sin\theta - \mu F_n\cos\theta$$

$$\frac{mv^2}{r} = F_n(\sin\theta - \mu\cos\theta)$$

$$\frac{mv^2}{r} = \frac{mg}{\mu\cos\theta + \sin\theta}(\sin\theta - \mu\cos\theta)$$

$$v = \sqrt{\frac{rg(\sin\theta - \mu\cos\theta)}{\mu\cos\theta + \sin\theta}}$$ ← The negative comes from this expression

Looking at FBDx, $F_{fx} > F_{nx}$

v = error.